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Abstract: This paper is intended to highlight the methodology of integrated planning for coordinating the supply chain in order to improve a system's overall performance. For that, a typical system is considered in which several products must be produced and distributed over several time periods. This work is done on a particular system, which includes several suppliers, raw material processing plants, finishing plants and customers, which are distributed across different geographical locations. Two main models have been developed, and they have been implemented in two different contexts. Various processes such as raw material procurement, capacity allocation, inventory, and distribution are integrated and optimized. More specifically, the following stages are considered: supply of raw material; production, at raw material processing plants, of product families; production, at finishing plants, of finished goods; and distribution of finished goods to customers. The finished goods are grouped into product families; production setups are incurred at the raw material processing plants for product families, and at the finishing plants for individual products. MIP (Mixed Integer Linear Program) formulations are utilized for optimizing the system. Test cases include both smaller-sized and larger-sized problems. Analysis is done to gain insights into the workings of the models and systems like these in general. Finally, the models are extended to include the transportation setups for distribution to raw material processing plants from suppliers and from finishing plants to customers, wherever required. Due to increased complexity and solution times for 100% optimality, possible meta-heuristics are proposed as future research.

Keywords: Supply Chain Management, Integrated Planning, Mixed Integer Programming

Résumé: Dans cet article, nous nous intéressons à la planification intégrée d'une chaîne d'approvisionnement dans le but d'en améliorer la performance globale. Nous étudions un système dans lequel plusieurs produits doivent être manufacturés et distribués sur plusieurs périodes de temps. Le réseau à l'étude comprend des fournisseurs, des usines de transformation de matières premières (où des familles de produits sont manufacturées), des usines de produits finis et des clients, qui sont tous localisés en des lieux différents. Différents processus tels que l'approvisionnement, l'allocation de capacité, la gestion des inventaires et la distribution sont intégrés et optimisés au moyen de la programmation linéaire à variables mixtes.

Mots-clés : Gestion de la chaîne d'approvisionnement, Planification intégrée, Programmation linéaire à variables mixtes.

Introduction

In recent times, the area of Supply Chain Management (SCM) has become a hot topic for researchers in Logistics, Production/Operations Management and Management Science all around the world; and the terms Supply Chain and Value Chain (Oliver & Webber, 1982, Porter, 1985) have gained significant popularity. SCM and Value Chain Management (VCM) are now considered as prime factors for success in business management. Although SCM has been defined in many different ways, the general consensus is that it is concerned with the integration of procurement, production, and distribution of commodities. Thus integrated planning is at the heart of any effectively managed supply chain. A few notable definitions are provided in the references (Shapiro, 2001; Simchi-Levi, 2003).

A huge body of literature is available on the various methodologies that researchers have considered for solving the hard combinatorial optimization problems arising in SCM. An earlier formulation of an integrated production-distribution problem was by Folie *et al.* (1976). Thomas *et al.* (1996) review many of those methodologies for a coordinated SCM; they divide them according to 3 operational scenarios: buyer-vendor coordination, production-distribution coordination and inventory-distribution coordination. Cohen *et al.* (1988) present a framework for linking decisions and performance in a production-distribution supply chain. Chandra *et al.* (1994) present a single plant, multi-product, multiple time periods model that integrates the production planning and vehicle routing decisions. In a somewhat similar situation as ours, Özdamar *et al.* (1999) present a hierarchical model for integrating a production-distribution system. Sox *et al.* (1996), using Langrangian relaxation, develop an algorithm for the finite horizon capacitated production planning problem. Chen *et al.* (1997) present an integrated methodlogy relevant to a steel manufacuring firm.

In this paper we generalize and extend earlier results obtained by Chen *et al.* (1997). We consider an integrated planning problem in which several products must be produced and distributed over several time periods. More specifically, we consider the following stages:

- Supply of raw material
- Production of product families at raw material processing plants
- Production of finished goods at finishing plants, and
- Distribution of finished goods to customers

We assume that the following data is available: demand for each customer, production rates, yields and costs at various stages of production and distribution, setup costs and times at various factories for various product families and for individual products, setup costs for transportation from supplier to raw material processing plants and from finishing plants to customers.

The model presented in Chen *et al.* was an LP. Here due to the presence of setups, we have to work with both continuous and 0-1 variables. Two different models have been studied; they have been implemented in two different contexts. In all, four different models have been formulated and developed. All the models have been formulated and solved with the commercial software LINGO™ on a Pentium-4 machine. We have tested our programs on a variety of problem instances. Some of

them (e.g. 3 time periods, 2 raw material processing plants, 3 finishing plants, 3 (corporate) customers, 3 product families, 6 products) have been solved to optimality. Larger ones have been solved to 1% optimality. Problem difficulty has been analyzed related to the size and tightness of the constraints and data.

The Network

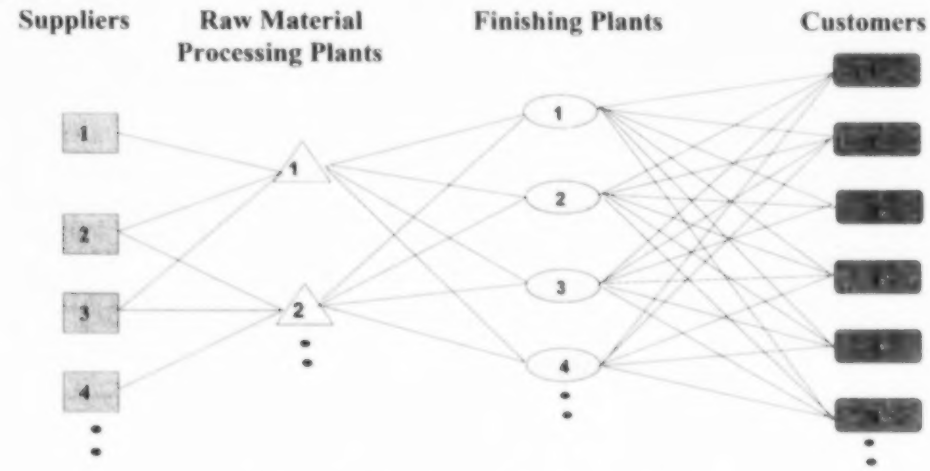

Figure 1: Structure of the Supply Chain

Figure 1 depicts the structure of the SC under study. Raw materials are shipped to the raw material processing plants (RMP), where they are transformed into various product families. These product families are further transformed into individual products at the finishing plants (FP). A simple example could be given by steel production, as in Chen *et al.* In that case the raw material could be scrap steel, which is sent to RMPs for conversion to different ingot types. One of the ingot types could be utilized to make utensils, while another could be used to make pipes. Products are made according to the customers' forecasted demand over the horizon, the inventory at hand and capacity restrictions. Customers can be thought of as regions, rather than individuals. Initially, the cost of transportation to these regions is assumed to be linear. Extensions are later formulated to represent fixed costs associated with transportation setups. In each time period of the planning horizon, available production capacities are limited in both RMPs and FPs. Therefore this resource must be judiciously apportioned among different product families and individual products in order to yield the highest profit.

2 An Integrated Planning Model

Raw material supply is limited; and the purchase price may vary from one supplier to the next. Raw material transportation cost may also be variable. There is a production rate for each product family and each individual product, measured in units/hour. There is also a transformation rate for each product family and each individual product. Selling prices of the finished products depend on customer regions so that the same product may have different selling prices in different regions.

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2.1 Modeling the System

We have developed two main models (1 and 2) and implemented them in two different settings (A and B). See Table 2. In the second formulation type, by appropriately defining the model variables, one is able to incorporate the provision for tracking the products. This helps manage various parts of inventories at the planning stage itself. Management policies regarding load sharing and enhancing the reliability of the supplier network can then be added on, based on the current optimal solution. Furthermore, maintenance schedules and procedures can also be evaluated at the planning stage itself. This will help in preventive maintenance and transferring loads to other manufacturing facilities based on the maintenance schedule.

In the first formulation type there is no provision for backtracking the products. This formulation applies to the general case where being able to trace back the inputs to a product is not a requirement.

In the formulations for setting A, the demand is being pooled for each finishing plant (FP). This particular case is relevant to a franchising environment where the demand from individual outlets is either aggregated and received at a distribution center (DC) from a dedicated plant, or at least globally managed by a central authority. The main advantage for systems such as these is that they allow risk pooling to take place. As a result, better quality of service can be achieved with lower inventory levels (Simchi Levi, 2003).

As far as situation B is concerned, the demand from each customer is to be met individually. Modeling for this type of scenario is slightly more complex and requires extra variables.

From these definitions it becomes evident, since the supply chain structure underlying the mathematical models is essentially the same, that models (A or B) of Type 1 and 2 should provide the same maximized (or minimized) objective function value, irrespective of the values of the decision variables..

2.2 Indices

In presenting the models the following general notation is used.
j = index of the raw material suppliers, $j \in \{1 \ldots J\}$
c = index of the raw material processing plants, $c \in \{1 \ldots C\}$
f = index of the finishing plants, $f \in \{1 \ldots F\}$
l = index of the customer regions, $l \in \{1 \ldots L\}$
m = index of the product families, $m \in \{1 \ldots M\}$
$n(m)$ = index of the product items in a product family, $n(m) \in \{1 \ldots N_M(M)\}$.
t = index of the time periods in a planning horizon, $t \in \{1 \ldots T\}$.

We will now compare the variables for the two main models. The production quantities calculated, such as $X_{fln(m)t}$ are continuous. The integrality in the models comes from the setup variables, such as S_{cmt} as defined in Table 1.

2.3 General Assumptions

The assumptions fall into four categories.

2.3.1 Material Supply and Transportation

a) Raw material from each supplier is limited in each time period. The associated purchasing cost may be different for each supplier. Initially the freight cost is assumed to be linear. Subsequently, extensions of the models incorporating transportation setups are also formulated.

2.3.2 Production

b) A setup is incurred whenever production moves from one family/product to another.
c) Each plant has a production capacity that is expressed as total available production time.

2.3.3 Inventory

d) An RMP only keeps inventory of the raw material, and therefore sends whatever it produces to the FPs. Similarly an FP only keeps inventory of the finished products, i.e. it transforms whatever it receives from the RMPs.
e) In the models of type 1, inventory at the RMPs is defined with respect to a particular plant and the time period, while for type 2 models, inventory at the RMPs is defined with respect to a particular plant, at a particular time period, of the raw material received from a particular supplier.

2.3.4 Demand

f) Demand has to be met in each time period. If it is not, then the programs will indicate infeasibility.
g) Demand is considered as deterministic.
h) For the situation A models, demand for product n, by customer l, is to be satisfied by plant f, at time t. For the situation B models, demand for product n, by customer l must be met at time t by any of the FPs either individually or combined.

3 Example and Analysis

Here, we present a numerical example, for the sake of illustration, of the integrated methodology presented in the previous sections. We consider three suppliers (J=3), two central plants (C=2), three finishing plants (F=3), three customers (L=3), three time periods (T=3), two product families (M=2) and three products in each family ($N(1)$=3, $N(2)$ = 3). The number of integer variables for this example is sixty-six (66). As we can see in Tables 4 and 5, the number of units in time periods t = 2 and t = 3 are different for the two models; however, the total quantities are the same; for Model 1, 9510 + 4600 + 11850 + 5300 = 31260; for Model 2, 6300 + 8700 + 10000 + 6260 = 31260. A similar phenomenon is observed for the quantities sent to the FPs by the RMPs. Table 6 gives the total profit (the objective function that is to be maximized) and its breakdown, for Models 1 and 2. The profit components are different in the two models but the total is the same. Similar agreements were observed for all the instances that were solved to optimality. Since it is unlikely that this occurred by chance, these agreements provide evidence of the correctness of both models. In that sense, we can say that Models 1 and 2 serve to validate one another.

We report on two instances for each model (see Table 3). Both are realistic. While the first instance is smaller, the other one is larger. Models 1A and 2A reflect situation A, while models 1B and 2B reflect situation B. Their parameters, along with the problem size, and the CPU times, are also given. Execution times vary, depending on the tightness of data and problem size. Moreover, as is true of all NP-hard problems, the solution times explode once the problem size increases, however modestly, beyond a certain threshold. For the former, models 1A and 2A give the same global optimum value, so do too models 1B and 2B. For the latter, models 1A and 2A again give the same optimal value. Models 1B and 2B, because of large solution times and the presence of a larger number of integer variables (110), are only solved to 1% optimality. The solution values are close to each other and their bounds agree, leading us to believe that if allowed to run for a long enough time, they may in the end provide us with the same global optimum value.

4 Extensions - Incorporation of Transportation Setups

Another level of decision-making can be incorporated to bring the models closer to practicality: this can be introduced by incorporating setup variables for transportation so as to deal with full and partial truckloads.These setups are important in evaluating the tradeoffs between transportation and inventory.

By analyzing the results (Table 7 and Table 8), it becomes evident that incorporating transportation setups lead to dissimilar, but agreeable results, when comparing models with and without setups. The dissimilarities between the results are on account of additional setups and as can be seen, the solution tries to "save" on the setup cost by utilizing a single supplier. This is expected and corroborates the modeling formulation. Also, we report the same objective function value for Models 1 and 2. This is because essentially Model 1 and Model 2 represent a same situation modeled in two different ways. Model 1B and 2B are solved to 1% optimality because of large solution times. The results are presented (Table 9).

5 Summary and Conclusion

In this paper, an integrated approach to a production-distribution system was formulated and discussed. Two models were created and implemented in two different settings. These models were studied using both smaller size and larger size problems. Smaller instances were solved to optimality. For larger instances, 1% optimal solutions were readily obtained. In view of inherent inaccuracies involved in data collection, solutions that are optimal up to 1% may in many cases be quite acceptable. With quickly obtainable sub-optimal (1%) solutions, an analyst can simulate several scenarios before making a final decision.

Furthermore, the original models were extended to include setups relating to transportation links between 2 nodes.

6 Future Research

The mixed integer programming formulations can be strengthened by the addition of valid inequalities, and meta-heuristics can be designed for providing "good" solutions to large instances of these problems. Also, integrated planning can be further generalized so as to include scheduling and vehicle routing. Furthermore, capacity restrictions can be added on the transportation side as well to evaluate individual tradeoffs associated with discrete transportation loads.

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8 Tables:

Table 1: Decision Variables for Situation A*

Model 1		Model 2	
Decision Variable	**Description**	**Decision Variable**	**Description**
$X_{fln(m)t}$	Amount of product n, of product family m, produced by FP f, for customer l, at time t.	$X_{cfln(m)t}$	Amount produced by FP f, of a product n, of product family m, for customer l, from the material sent to f, by RMP c (As is evident, there is an additional index in the variable).
X_{cfmt}	Amount of product family m, produced by RMP c, for FP f, at time t.	X_{jcfmt}	Amount of product family m, produced by RMP c, from material by supplier j, for FP f, at time t.
$I_{fln(m)t}$	Inventory at finishing factory f for customer l of product family m, for product n at time t.	$I_{fln(m)t}$	Inventory at finishing factory f for customer l of product family m, for product n at time t.
I_{ct}	Inventory of raw material at RMP c at time period t.	I_{jct}	Inventory of raw material at RMP c at time period t sent by supplier j.
W_{jct}	Amount of raw material purchased from supplier j at time t for RMP c.	W_{jct}	Amount of raw material purchased from supplier j at time t for RMP c.
S_{cmt}	Binary variable signifying a setup for the product family m at RMP c at time t (0 = no setup 1= setup present).	S_{cmt}	Binary variable signifying a setup for the product family m at RMP c at time t.
$S_{fn(m)t}$	Binary variable signifying a setup for the product n of family m at FP f at time t.	$S_{fn(m)t}$	Binary variable signifying a setup for the product n of family m at FP f at time t.

***: There would be a slight change in the decision variables corresponding to situation B**

Table 2: Different Types of Models Developed

Situation / Type	**A** (Demand is pooled for each FP)	**B** (Demand of individual customers is known)
1 (No provision is made to trace back the product)	Model 1A	Model 1B
2 (Products can be traced back)	Model 2A	Model 2B

Table 3: Report on two Instances

	Index	Model 1A	Model 2A	Model 1B	Model 2B
Example 1	j	3	3	3	3
	c	2	2	2	2
	f	3	3	3	3
	l	3	3	3	3
	m	2	2	2	2
	n	6	6	4	4
	t	3	3	3	3
	Size (constraints X variables)	289 X 406	480 X 809	181 X 286	324 X 627
	Integer variables	66	66	48	48
	Time	5 sec	9 sec	120 sec	60 sec
Example 2	j	3	3	3	3
	c	2	2	2	2
	f	3	3	3	3
	l	3	3	3	3
	m	4	4	2	2
	n	8	8	6	6
	t	5	5	5	5
	Size	643 X 979	1108 X 1997	383 X 652	708 X 1477
	Integer variables	160	160	110	110
	Time (*: 1% optimal solution)	260 sec	45 min	40 sec * Best IP: 1.458 IP Bound: 1.463	50 sec * Best IP: 1.460 IP Bound: 1.465

Table 4: Model 1- Production Quantities at the RMPs (Smaller Instance)

c_1t_1: Total Raw Material (10000 Units)			c_1t_2 (9510 Units)			c_1t_3 (11850 Units)		
j_1=10000	j_2=0	j_3=0	j_1=4510	j_2=0	j_3=5000	j_1=10000	j_2=1850	j_3=0
c_2t_1 (10000 units)			c_2t_2 (4600 units)			c_2t_3 (5300 units)		
j_1=0	j_2=0	j_3=10000	j_1=490	j_2=4110	j_3=0	j_1=0	j_2=300	j_3=5000

Table 5: Model 2- Production Quantities at the RMPs (Smaller Instance)

c_1t_1 (10000 Units)			c_1t_2 (6300 Units)			c_1t_3 (10000 Units)		
j_1=10000	j_2=0	j_3=0	j_1=1300	j_2=0	j_3=5000	j_1=10000	j_2=0	j_3=0
c_2t_1 (10000 Units)			c_2t_2 (8700 Units)			c_2t_3(6260 Units)		
j_1=0	j_2=0	j_3=10000	j_1=3700	j_2=5000	j_3=0	j_1=0	j_2=1260	j_3=5000

Table 6: Breakdown of Total Profit (Smaller Instance)

Model 1		Model 2	
Type of Cost	**Description**	**Type of Cost**	**Description**
Profit	Total profit from the products = 3,516,030.	*Profit*	Total profit from the products = 3,516,030.
Raw material purchase cost	Total cost of raw material purchased in the time horizon = 119,760.	*Raw material purchase cost*	Total cost of raw material purchased in the time horizon = 126,180.
Raw material transportation Cost	Cost of transporting the raw material to the RMPs in the time horizon = 117,800.	*Raw material and product family transportation cost*	Cost of transporting the product families and the raw material that is used to manufacture them, in the time horizon = 178,380.
Product family transportation cost	Cost of transporting product family m, from the RMPs to the FPs in the time horizon = 68,780.	*(See above)*	
Finished product transportation cost	= 65,840.	*Finished product transportation cost*	= 65,840.
Setup cost at RMPs	Setup costs incurred at the RMPs in the time horizon = 300.	*Setup cost at RMPs*	Setup costs incurred at the RMPs in the time horizon = 300.
Setup cost at FPs	Setup costs incurred at the FPs in the time horizon = 730.	*Setup cost at FPs*	Setup costs incurred at the FPs in the time horizon = 730.
Inventory cost at RMPs	Inventory costs incurred at the RMPs in the time horizon = 0.	*Inventory cost at RMPs*	Inventory costs incurred at the RMPs in the time horizon = 1780.
Inventory cost at FPs	Inventory costs incurred at the FPs in the time horizon = 6,520.	*Inventory cost at FPs*	Inventory costs incurred at the FPs in the time horizon = 6,520.

Table 7: Model 1- Raw Material Quantities sent at the RMPs (Without Transportation Setups)

c_1t_1 (10000 Units)			c_1t_2 (5410 Units)			c_1t_3 (10000 Units)		
j_1=10000	j_2=0	j_3=0	j_1=5000	j_2=410	j_3=0	j_1=10000	j_2=0	j_3=0
c_2t_1 (10000 Units)			c_2t_2 (8700 Units)			c_2t_3(7150 Units)		
j_1=0	j_2=0	j_3=10000	j_1=0	j_2=3700	j_3=5000	j_1=0	j_2=2150	j_3=5000

Table 8: Model 1- Raw Material Quantities sent at the RMPs (With Transportation Setups)

c_1t_1 (10000 Units)			c_1t_2 (5000 Units)			c_1t_3 (10000 Units)		
j_1=10000	j_2=0	j_3=0	j_1=5000	j_2=0	j_3=0	j_1=10000	j_2=0	j_3=0
c_2t_1 (10000 Units)			c_2t_2 (9110 Units)			c_2t_3(7150 Units)		
j_1=0	j_2=0	j_3=10000	j_1=0	j_2=4110	j_3=5000	j_1=0	j_2=2150	j_3=5000

Table 9: Summary Report on Solutions After Incorporation of Transportation Setups

	Index	Model 1A	Model 2A	Model 1B	Model 2B
Example	Size (constraints X variables)	335 X 452	521 X 848	227 X 336	371 X 674
	Integer variables	111	111	93	93
	Time	15 sec	40 sec	120 sec*	360 sec*

Appendix 1

Parameters Utilized in the Formulations for Setting A:

1. RC_{jct} = Unit purchasing cost of raw material: from supplier j to RMP c at time t.
2. TR_{jct} = Unit transportation cost o f raw material from j to the RMP c, at time t
3. FC_{cm} = Fixed cost for manufacturing at RMP c for product m.
4. VC_{cm} = Variable cost for manufacturing at RMP c for product m.
5. CSC_{cm} = Unit cost associated with the setup at RMP c for product family m.
6. IC_{ct} = Unit inventory cost at RMP c at time t.
7. TS_{cfmt} = Unit transportation cost of product m from RMP c to FP f at time t.

8. $FF_{fn(m)}$ = Fixed cost for the manufacturing of product n at FP f.
9. $VF_{fn(m)}$ =Variable cost for the manufacturing of product n at FP f.
10. $CSF_{fn(m)}$ = Unit cost associated with the setup at FP f for product n of family m.
11. $IF_{fln(m)t}$ = Unit inventory cost at FP f for customer l of product n at time t.
12. $TF_{fln(m)t}$ = Unit transportation cost of product n from FP f to customer l at time t.

13. $PR_{fln(m)t}$ = Unit selling price of product n to customer l produced by FP f at time t.
14. PS_m = Production rate of product family m: raw material unit/time unit.
15. $PF_{n(m)}$ = Production rate of finished product n: product family unit/time unit.
16. YM_m = Transformation rate for product family m: product family unit/raw material unit.
17. $YN_{n(m)}$ = Transformation rate for finished product n: finished product unit/product family unit.
18. KJ_{jt} = Capacity of supplier j at time t.
19. KC_{ct} = Available production capacity (time) at RMP c at time t.
20. KF_{ft} = Available production capacity (time) at FP f at time t.
21. SC_{cm} = Setup time at RMP c for a product family m.
22. $SF_{fn(m)}$ = Setup time at FP f for a product n of family m.
23. $DC_{fln(m)t}$ = Demand for product n by customer l to be satisfied by FP f at time t.

Additional Parameters Defined for Model 2

24. TS_{jcfmt} = Unit combined cost of raw material transportation (from supplier j to RMP c) and product family transportation (from RMP c to FP f).

Note that we assume that the raw material inventory does not depend on the supplier. Therefore no additional parameter is needed for the calculation of inventory cost for Model 2.

Mixed Integer Formulation for Models Representing Situation A

1. Raw Material Supply: Amount of raw material purchased by all the RMPs from supplier j at time period t should be less than its capacity, i.e.

$$\sum_{c=1}^{C} W_{jct} \le KJ_{jt} \qquad \forall j, \forall t$$

2. Inventory Balance Equations for Raw Material
   For Model 1:

$$\sum_{j=1}^{j=J} W_{jct} + I_{ct} = \sum_{f=1}^{f=F} \sum_{m=1}^{m=M} (X_{cfmt}/YM_m) + I_{c(t+1)} \qquad \forall c, \forall t$$

   For Model 2:

$$\sum_{j=1}^{j=J} (W_{jct} + I_{jct}) = \sum_{j=1}^{j=J} \sum_{f=1}^{f=F} \sum_{m=1}^{m=M} (X_{jcfmt}/YM_m) + I_{jc(t+1)} \qquad \forall c, \forall t$$

3. Production Capacity in the RMPs: Production at RMP c is constrained by the available production capacity (time) at time period t, i.e.
   For Model 1:

$$\sum_{f=1}^{f=F} \sum_{m=1}^{m=M} (X_{cfmt}/YM_m)/PS_m + \sum_{m=1}^{m=M} S_{cmt} \times SC_{cm} \le KC_{ct} \qquad \forall c, \forall t$$

For Model 2:

$$\sum_{j=1}^{j=J} \sum_{f=1}^{f=F} \sum_{m=1}^{m=M} (X_{jcfmt}/YM_m)/PS_m + \sum_{m=1}^{m=M} S_{cmt} \times SC_{cm} \le KC_{ct} \qquad \forall c, \forall t$$

4. Production Capacity in the FPs:
   For Model 1:

$$\sum_{l=1}^{l=L} \sum_{n=1}^{n=N} (X_{fln(m)t}/YN_{n(m)})/PF_{n(m)} + \sum_{n=1}^{n=N} S_{fn(m)t} \times SF_{fn(m)} \le KF_{ft} \qquad \forall f, \forall t$$

   For Model 2:

$$\sum_{c=1}^{C} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} (X_{cfln(m)t}/YN_{n(m)})/PF_{n(m)} + \sum_{n=1}^{n=N} S_{fn(m)t} \times SF_{fn(m)} \le KF_{ft} \qquad \forall f, \forall t$$

5. Linking Product-families and Finished Products: The total production amount of finished product *n (m)* is directly related to the production of product family *m* at a time period *t*, i.e.
   For Model 1:

$$\sum_{l=1}^{l=L} \sum_{n=1}^{n=N} (X_{fln(m)t}/YN_{n(m)}) = \sum_{c=1}^{c=C} X_{cfmt} \qquad \forall f, \forall m, \forall t$$

   For Model 2:

$$\sum_{l=1}^{l=L} \sum_{n=1}^{n=N} (X_{cfln(m)t}/YN_{n(m)}) = \sum_{j=1}^{j=J} X_{jcfmt} \qquad \forall c, \forall f, \forall m, \forall t$$

6. Inventory Balance Equations for the Finished Products:
   For Model 1:

$$X_{fln(m)t} + I_{fln(m)t} = DC_{fln(m)t} + I_{fln(m)(t+1)} \qquad \forall f, \forall l, \forall m, \forall n, \forall t$$

   For Model 2:

$$\sum_{c=1}^{C} (X_{cfln(m)t}) + I_{fln(m)t} = DC_{fln(m)t} + I_{fln(m)(t+1)} \qquad \forall f, \forall l, \forall m, \forall n, \forall t$$

7. Production/Setup Constraints: Production for a particular item (family *m)* or (product *n)* requires a setup at a plant (RMP *m*) or (FP *f*). Let M be a suitably large number. Then,

For Model 1:

$$\sum_{f=1}^{f=F} X_{cfmt} \le M \times S_{cmt} \qquad \forall c,\ \forall m,\ \forall t$$

$$\sum_{l=1}^{l=L} X_{fln(m)t} \le M \times S_{fn(m)t} \qquad \forall f,\ \forall m,\ \forall n,\ \forall t$$

For Model 2:

$$\sum_{j=1}^{j=J} \sum_{f=1}^{f=F} X_{jcfmt} \le M \times S_{mct} \qquad \forall c,\ \forall m,\ \forall t$$

$$\sum_{c=1}^{C} \sum_{l=1}^{l=L} X_{cfln(m)t} \le M \times S_{fn(m)t} \qquad \forall f,\ \forall n,\ \forall t$$

8. Ending Inventory Constraint: The finished good inventory at each FP at the end of the time horizon is assumed to be 0.

For Model 1 at FP:

$I_{fln(m)(T+1)} = 0$, where T is the ending horizon $\qquad \forall f,\ \forall l,\ \forall n$

Similar constraints are defined for the RMPs and for Model 2.

Objective Function: The objective in solving the planning problem is to maximize the total net profit, which can be expressed as:

Total net profit = Total revenue – Total cost.

For Model 1:

$$\text{Total } REVENUE = \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} PR_{fln(m)t} \times X_{fln(m)t}$$

For Model 2:

$$REVENUE = \sum_{c=1}^{C} \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} PR_{fln(m)t} \times X_{cfln(m)t}$$

The costs are calculated as indicated below:

- Raw Material Purchasing Cost: For both models it would be the same,

$$RMCOST = \sum_{j=1}^{j=J} \sum_{c=1}^{c=C} \sum_{t=1}^{t=T} (RC_{jct} \times W_{jct})$$

- Total Fixed Cost

It is assumed to be dependent on the total quantity of material produced in a particular time period. Therefore it can be represented as,
For Model 1:

$$FIXCOST = \sum_{j=1}^{j=J} \sum_{c=1}^{c=C} \sum_{m=1}^{m=M} \sum_{t=1}^{t=T} FC_{cm} \times X_{jcmt} + \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} FF_{fn(m)} \times X_{fln(m)t}$$

For Model 2:

$$FIXCOST = \sum_{j=1}^{j=J} \sum_{c=1}^{c=C} \sum_{m=1}^{m=M} \sum_{f=1}^{f=F} \sum_{t=1}^{t=T} FC_{cm} \times X_{jcmft} + \sum_{c=1}^{c=C} \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} FF_{fn(m)} \times X_{cfln(m)t}$$

- Total Variable Cost:
For Model 1:

$$VARCOST = \sum_{j=1}^{j=J} \sum_{c=1}^{c=C} \sum_{m=1}^{m=M} \sum_{t=1}^{t=T} VC_{cm} \times X_{jcmt} + \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} VF_{fn(m)} \times X_{fln(m)t}$$

For Model 2:

$$VARCOST = \sum_{j=1}^{j=J} \sum_{c=1}^{c=C} \sum_{m=1}^{m=M} \sum_{f=1}^{f=F} \sum_{t=1}^{t=T} VC_{cm} \times X_{jcmft} + \sum_{c=1}^{c=C} \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} VF_{fn(m)} \times X_{cfln(m)t}$$

- Raw Material Transportation Cost for Model 1:

$$RMTRPCST = \sum_{c=1}^{c=C} \sum_{j=1}^{j=J} \sum_{t=1}^{t=T} TR_{jct} \times W_{jct}$$

- Product Family Transportation Cost from RMPs to FPs for Model 1:

$$SFCPTRCT = \sum_{c=1}^{c=C} \sum_{f=1}^{f=F} \sum_{m=1}^{m=M} \sum_{t=1}^{t=T} TS_{cfmt} \times X_{cfmt}$$

- For Model 2 the costs of raw material transportation and product family transportation are aggregated and calculated as follows. (See Table 6)

$$RMTRPCST = \sum_{c=1}^{c=C} \sum_{j=1}^{j=J} \sum_{f=1}^{f=F} \sum_{m=1}^{m=M} \sum_{t=1}^{t=T} TS_{jcfmt} \times X_{jcfmt}$$

- Finished Product Transportation Cost from the FPs to the Customers:
For Model 1:

$$FNTRPCST = \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} TF_{fln(m)t} \times X_{fln(m)t}$$

For Model 2:

$$FNTRPCST = \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} TF_{fln(m)t} \times \sum_{c=1}^{c=C} X_{cfln(m)t}$$

- Setup Costs at RMPs: Same for both models,

$$SETCPCST = \sum_{c=1}^{c=C} \sum_{m=1}^{m=M} \sum_{t=1}^{t=T} S_{cmt} \times CSC_{cm}$$

- Setup Costs at FPs: Same for both models,

$$SETFFCST = \sum_{f=1}^{f=F} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} S_{fn(m)t} \times CSF_{fn(m)}$$

- Inventory Costs at RMPs:
  For Model 1:

$$INVCPCST = \sum_{c=1}^{c=C} \sum_{t=1}^{t=T} I_{ct} \times IC_{ct}$$

For Model 2:

$$INVCPCST = \sum_{j=1}^{j=J} \sum_{c=1}^{c=C} \sum_{t=1}^{t=T} I_{jct} \times IC_{ct}$$

- Inventory Costs at FPs: Same for both models,

$$INVFFCST = \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} I_{fln(m)t} \times CI_{fln(m)t}$$

Thus from the above expressions:
Profit = REVENUE – (RMCOST + FIXCOST + VARCOST + RMTRPCST + SFTRPCST + FNTRPCST + SETCPCST + SETFFCST + INVCPCST + INVFFCST)

And the objective function is
MAXIMIZE *(Profit)*

Appendix 2

Parameters Used for Setting B Only:

1. $DC_{ln(m)t}$ = Demand for product *n* by customer *l* at time *t*.

2. $PR_{fln(m)t}$ = Unit selling price of product n produced by FP f to customer l at time t.

Variables Used for Setting B Only:

1. $X_{fn(m)t}$ = Production at FP f of a product n of family m at time period t.
2. $Y_{fln(m)t}$ = Shipment of product $n(m)$ from FP f to customer l at time period t.

Mixed Integer Formulation for Models Representing Situation B

Only the modified constraints are presented.

- Inventory Balance Constraints at the FP
  For Model 1:

$$X_{fn(m)t} + I_{fn(m)t} = Y_{fln(m)t} + I_{fn(m)(t+1)} \qquad \forall f,\ \forall l,\ \forall m,\ \forall n,\ \forall t$$

  For Model 2:

$$\sum_{c=1}^{C} (X_{cfln(m)t}) + I_{fln(m)t} = Y_{fln(m)t} + I_{fln(m)(t+1)} \qquad \forall f,\ \forall l,\ \forall m,\ \forall n,\ \forall t$$

- Constraints for the Customer Demand
  $Y_{fln(m)t}$ represents the quantity transported to the customers from the FPs. Therefore we have;
  For both Model 1 and Model 2:

$$\sum_{f=1}^{f=F} Y_{fln(m)t} = DC_{ln(m)t} \qquad \forall l,\ \forall n,\ \forall t$$

Other minor modifications are appropriately made in the cost functions.
MAXIMIZE *(Profit)*.

Appendix 3

Additional Parameters for Transportation Setups:

1. $CTR_{j,c,t}$ = Cost of Transportation setup for link between supplier j, RMP c at time t.
2. CTF_{flt} = Cost of Transportation setup for link between FP f, customer l at time t.

Variables Used for Transportation Setups:

3. T_{jct} = Binary variable signifying a setup for the transportation link between supplier j and RMP c at time t (0 = no setup 1= setup present).
4. T_{flt} = Binary variable signifying a setup for the transportation link between FP f and customer l at time t (0 = no setup 1= setup present).

Mixed Integer Formulation for Models Representing Transportation Setups

Only additional constraints and modified equations are presented.

- Transportation Setup Constraint for RMPs (same for all Models): Transportation link between a supplier j and an RMP c requires a setup. Using Big number M again,

For Model 1:

$$W_{jct} \leq M \times T_{jct} \qquad \forall j, \forall c, \forall t$$

- Transportation Setup Constraint for FPs: Transportation link between an FP f and a customer l requires a setup. Using Big number M again,
For Model 1A:

$$\sum_{n=1}^{N} X_{fln(m)t} \leq M \times T_{flt} \qquad \forall f, \forall l, \forall t$$

For Model 2A:

$$\sum_{c=1}^{C} \sum_{n=1}^{N} X_{cfln(m)t} \leq M \times T_{flt} \qquad \forall f, \forall l, \forall t$$

For Model 1B and 2B:

$$\sum_{n=1}^{N} Y_{fln(m)t} \leq M \times T_{flt} \qquad \forall f, \forall l, \forall t$$

- Raw Material Transportation Cost (same for all Models): This will modify slightly to represent additional setups

$$RMTRPCST = \sum_{c=1}^{c=C} \sum_{j=1}^{j=J} \sum_{t=1}^{t=T} (TR_{jct} \times W_{jct}) + (CTR_{jct} \times T_{jct})$$

- Finished Product Transportation Cost from the FPs to the Customers: Similarly this calculation will be modified to reflect the additional setups
For Model 1A:

$$FNTRPCST = \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} TF_{fln(m)t} \times X_{fln(m)t} + \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{t=1}^{t=T} (TF_{flt} \times T_{flt})$$

For Model 2A:

$$FNTRPCST = \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} TF_{fln(m)t} \times \sum_{c=1}^{c=C} X_{cfln(m)t} + \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{t=1}^{t=T} (TF_{flt} \times T_{flt})$$

For both Model 1B and Model 2B:

$$FNTRPCST = \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{n=1}^{n=N} \sum_{t=1}^{t=T} TF_{fln(m)t} \times Y_{fln(m)t} + \sum_{f=1}^{f=F} \sum_{l=1}^{l=L} \sum_{t=1}^{t=T} (TF_{flt} \times T_{flt})$$